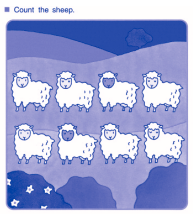
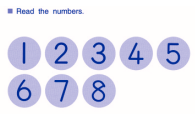
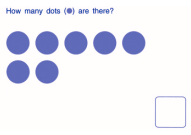
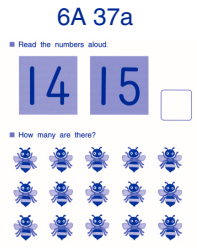
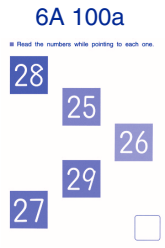
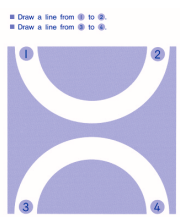
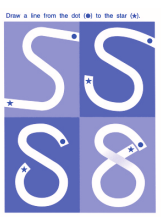
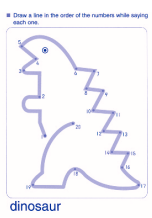
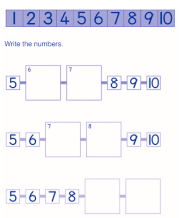
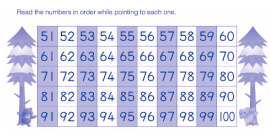
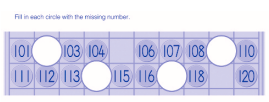
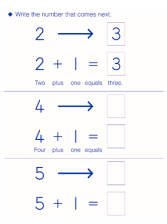
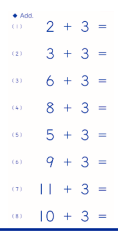
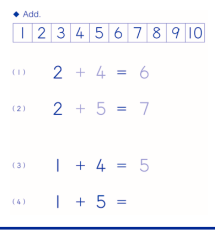
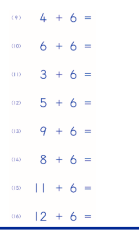
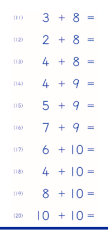
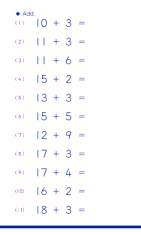
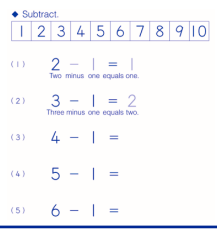
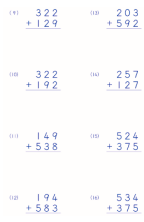
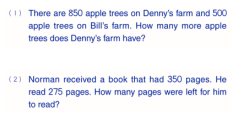
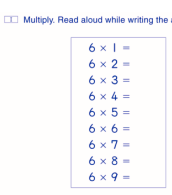
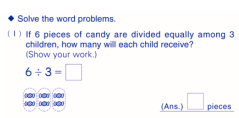
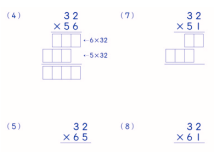
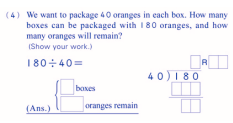
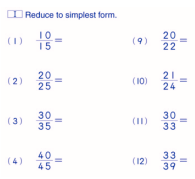
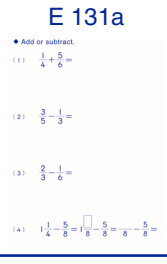
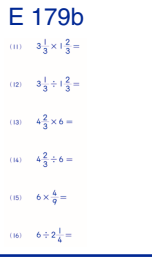
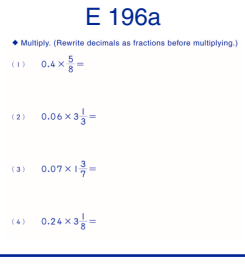
KUMON® DESCRIPTION OF STUDY LEVELS - MATH

Level	Samples	Highlights
7A	7A 76a  7A 76b  7A 161a 	Students count up to 10 pictures and dots individually and as a group. Mastery is gradual and the eventual goal is for students to be able to say the total number of objects in each group without counting.
6A	6A 37a  6A 100a  6A 173a 	Students count up to 30 using pictures and numbers. Gradually, students learn to recognize groups of up to 20 dots without counting them individually.
5A	5A 41a  5A 147a  5A 187a  dinosaur	Students learn to use a pencil through line tracing exercises, beginning with short lines and advancing to long curved lines. The curved lines gradually take the shape of large numbers. This develops the fine motor skills needed to trace and write numbers independently and teaches the natural stroke order required for number formation. Students also develop their concentration ability and learn to recite numbers up to 50.
4A	4A 40a 4A 79a  4A 191b 	Students learn to write numbers up to 50. Students deepen their understanding of the number sequence through writing consecutive numbers and filling in the blanks in number tables, number boards, and complete-the-sequence problems. By the end of the level, students learn to read up to 100.
3A	3A 69b  3A 74a  3A 195a 	Students continue to enhance their understanding of the sequence of numbers as well as the number-writing skills that they developed in Level 4A. Students are introduced to addition in Level 3A. At first, they master +1, +2, through to +3 individually. The last 20 sheets of this level are dedicated to addition questions from +1 to +3.
2A	2A 31a  2A 78b  2A 200b 	In this level, students further develop basic mental calculation skills in addition through a sequential study of adding 4 through adding 10. It is very important that students master the contents of this level for smooth progress in subsequent levels. Level 2A aims to develop the concentration ability and work skills necessary for Level A.
A	A 68a  A 81a  A 191a 	Level A continues horizontal addition with larger numbers. Mastering addition ensures a smooth introduction to subtraction in this level. This level aims to develop mental calculations, while strengthening students' concentration ability and their work skills.
B	B 80b  B 187a B 199a 	This level teaches vertical addition and subtraction. Throughout it, students will encounter their first word problems in Kumon. This level draws on the advanced mental calculation skills learned in previous levels when students "carry" in addition questions and "borrow" in questions involving subtraction. Mastery of Level B greatly reduces errors in multiplication and division in Levels C and D.
C	C 32a  C 121a  C 200b 	Students master the multiplication tables by practicing until they can answer immediately. Next, students learn up to 4-digit by 1-digit multiplication with mental carryovers. Once multiplication is mastered, simple division by 1-digit is introduced. Students who have developed good mental calculation ability will not have to write division steps.
D	D 11b  D128b  D166a 	Students learn double digit multiplication before advancing to long division. In this challenging section, students develop estimation skills that will be necessary for future fraction work. Once students' ability to work with all 4 arithmetic operations is confirmed, they begin to study fractions, learning to reduce using the Greatest Common Factor.
E	E 131a  E 179b  E 196a 	Students learn to add, subtract, multiply, and divide fractions. Proper intermediate steps are emphasized. At the end of the level, students learn basic fraction/decimal conversions.

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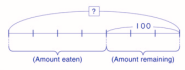
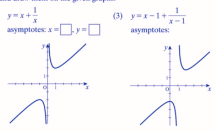
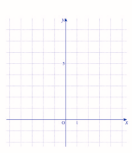
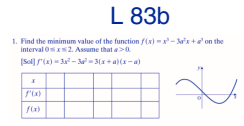
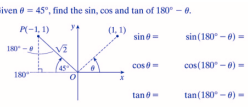
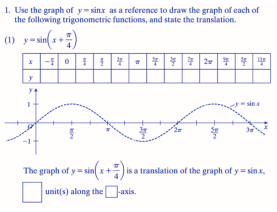
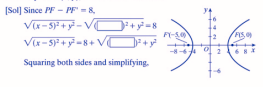
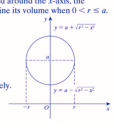
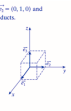
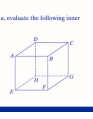
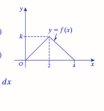
Level	Samples	Highlights
F	**F 108a** (2) $5+4\times6-12=$ (3) $3-\frac{1}{2}\div\frac{1}{5}=$ **F 151b** (6) $2\times\square=12$ **F 165b** (3) Ted ate $\frac{4}{7}$ of a container of ice cream. 100 g of ice cream remain. How many grams of ice cream were there originally?  (Show your work.) (Ans.) _____ unit **F 187b** (11) $265\div10=$ (12) $265\div100=$ (13) $265\div1000=$ (14) $265\div10000=$	Students continue calculations with fractions, now employing the order of operations. Level F contains a challenging section of word problems, as well as more work with decimals.
G	**G 21a** (3) $5-4=$ (4) $5-5=$ (5) $5-6=$ (6) $5-7=$ **G 80b** (10) $\left(\frac{1}{2}\right)^3-\left(\frac{1}{2}\right)^2=$ (11) $4^3+(-5)^3=$ (12) $4^3-(-5)^3=$ **G 162a** (1) $5x+4=3x+8$ [Sol] $5x-\square=8-\square$ (2) $-3x+6=-5x-2$ [Sol] $-3x+\square=-2-\square$	Students are introduced to positive and negative numbers, as well as to basic algebra. Students use their previously learned four operations skills to master linear equations. A word problem set rounds off the level, allowing students to apply everything they have learned in Level G.
H	**H 28a** (1) $x=3(x-a)$ (2) $a(x-2)=b$ (3) $c(1+bx)=a$ (4) $\frac{2}{3}(1+x)=a$ **H 76b** (2) $\begin{cases}\frac{-x-3}{5}=\frac{y-7}{2}\\-11x=13y\end{cases}$ (3) $\begin{cases}-3y+x=0\\-\frac{y}{2}=\frac{x}{3}-3\end{cases}$ **H 124b** (13) $3x+3<5x$ (14) $-5x+6>-2x-3$	Students will learn to solve simultaneous linear equations in two to four variables. Concepts of numerical and algebraic value are strengthened. Students are introduced to transforming equations, inequalities, functions and graphs.
I	**I 37a** (5) $x^2y^2-12xy+36=(xy-\square)^2$ (6) $a^2x^2+6ax+9=$ (7) $4x^2y^2+12xy+9=$ **I 99b** (9) $2\sqrt{18}+\sqrt{50}=\square\sqrt{2}+\square\sqrt{2}=$ (10) $\sqrt{48}+5\sqrt{12}=$ (12) $2\sqrt{12}-\sqrt{27}=$ **I 134a** (2) $-2x^2-7x-3=0$ (4) $-3x^2-7x+5=0$	This level thoroughly reviews Levels G and H and introduces factorization. Factorization is an essential skill to advance to square roots and quadratic equations, also covered in the level. The level concludes with advanced topics in geometry, specifically related to the Pythagorean Theorem.
J	**J 30a** (3) $(a-2b)(3x-5y)+(2b-a)(x-y)=$ (4) $4xy^2(3y-x)-2x^2(x-3y)^2=$ **J 115b** (1) $(2x-y)+(y-x)i=1+3i$ (2) $(1+i)x-(1-2i)y=1+4i$ $(\quad)+(\quad)i=1+4i$ **J 166b** 2. When a polynomial $P(x)$ is divided by $x-3$, the remainder is 3. When it is divided by $x+4$, the remainder is -4. Find the remainder when $P(x)$ is divided by $(x-3)(x+4)$. **J 196b** 2. Prove the following inequalities. (1) Given $a+b=1$, prove the inequality $a^2+b^2>ab$. (2) Given $a+b=1$, prove the inequality $a+b>ab$.	Concepts learned through Level I are expanded and reinforced. Students are introduced to advanced factoring methods, complex numbers, the discriminant, and the *Factor* and *Remainder* theorems. At the end of Level J, students conduct proofs of algebraic equalities and inequalities.
K	**K 43b** 2. Draw the graph of the given quadratic function, find the maximum and minimum values, and then, state the range, (the set of y values) that satisfies the domain. (1) $y=x^2+3\ (-1\le x\le2)$ (3) $y=-x^2+2x+1\ (-1\le x\le2)$ Maximum value: $\square$, at $x=\square$ Minimum value: $\square$, at $x=\square$ Range: $\square\le y\le\square$ **K 138a** 1. For each given fractional function, find the equations of the asymptotes and draw them on the given graphs. (1) $y=x+\frac{1}{x}$ (3) $y=x-1+\frac{1}{x-1}$  asymptotes: $x=\square$, $y=\square$ asymptotes: **K 183a** Draw the graph of each of the following exponential functions. (1) $y=2^{x-1}$ (2) $y=\left(\frac{1}{2}\right)^{x-1}$ 	Students acquire the basic properties of functions, through a thorough study of quadratic functions. Level K introduces higher degree, fractional, irrational and exponential functions and their corresponding graphs. The skills developed here will help ease students into the calculus exercises of Level L.
L	**L 14a** Evaluate the following expressions. (1) $\log_3 36-4\log_9 30+16\log_{81}\sqrt{15}$ = (2) $\log_2 6\cdot\log_3 6-(\log_3 3+\log_3 2)$ = (3) $\log_2 14\cdot\log_7 14-(\log_7 7+\log_7 2)$ = **L 42a** Find the following limit values. (1) $\lim_{x\to-2}\frac{x^2-x-6}{x+2}=$ (2) $\lim_{x\to2}\frac{x^2+2x-8}{x-2}=$ (3) $\lim_{x\to-1}\frac{x^3+1}{x+1}=$ **L 83b** 1. Find the minimum value of the function $f(x)=x^3-3a^2x+a^3$ on the interval $0\le x\le2$. Assume that $a>0$. [Sol] $f'(x)=3x^2-3a^2=3(x+a)(x-a)$  **L 145a** 1. Complete the following exercises. (1) Find the area, S_1, of the region enclosed by $y=-x^2+5x-2$, the x-axis, $x=1$ and $x=3$. [Sol] $S_1=\int_1^3(-x^2+5x-2)dx$ =	Students begin Level L by studying logarithmic functions, and are led into the beginning of calculus. Students study basic differentiation and definite and indefinite integrals. The level concludes with an analysis of the applications of integration, including areas, volumes, velocity and distance.
M	**M 22a** 1. Given $\theta=45°$, find the sin, cos and tan of $180°-\theta$.  $\sin\theta=$ $\sin(180°-\theta)=$ $\cos\theta=$ $\cos(180°-\theta)=$ $\tan\theta=$ $\tan(180°-\theta)=$ **M 73a** 1. Use the graph of $y=\sin x$ as a reference to draw the graph of each of the following trigonometric functions, and state the translation. (1) $y=\sin\left(x+\frac{\pi}{4}\right)$  The graph of $y=\sin\left(x+\frac{\pi}{4}\right)$ is a translation of the graph of $y=\sin x$, $\square$ unit(s) along the $\square$-axis. **M 125a** Convert each of the following sums or differences into products, using the Sum-to-Product Formulas and trigonometric properties. Ex. $\sin x+\sin(x-2y)=2\sin\frac{x+(x-2y)}{2}\cos\frac{x-(x-2y)}{2}$ $=2\sin(x-y)\cos y$ (1) $\cos(6x-2y)+\cos y=$ **M 198b** 3. Let Q be the point of tangency of the line drawn from point $P(3,0)$ to the circle $x^2+y^2=4$, and find the length of PQ using the Pythagorean Theorem. 	In Level M, students begin by studying the basics of trigonometric functions, graphs and inequalities. Students are then introduced to more advanced trigonometric topics including the *Addition Theorem*. At the end of Level M, students study analytic geometry.
N	**N 6a** 1. Given the fixed points $F(5,0)$ and $F'(-5,0)$, obtain the equation of the locus of point $P(x,y)$, such that $PF-PF'=8$. [Sol] Since $PF-PF'=8$, $\sqrt{(x-5)^2+y^2}-\sqrt{(\quad)^2+y^2}=8$ $\sqrt{(x-5)^2+y^2}=8+\sqrt{(\quad)^2+y^2}$ Squaring both sides and simplifying,  $\frac{x^2}{16}-\frac{y^2}{\square}=1$ **N 35a** 2. Obtain the first term and common difference of the arithmetic sequence whose 6^{th} term is -23 and 17^{th} term is 32. **N 123a** In each exercise, state whether the given series converges or diverges. If it converges, determine the sum. (1) $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots\cdots+\frac{1}{1+2+\cdots\cdots+n}+\cdots\cdots$ **N 157a** Find the limit values of the following functions. (2) $\lim_{x\to0}\frac{\sqrt{2-\cos x}-1}{3x^2}$ **N 189a** 1. Obtain the first and second order derivatives of each of the following functions. (1) $y=\left(\frac{1}{2x+1}\right)^2$	Students begin Level N by studying loci and quadratic inequalities. They then study arithmetic, geometric, infinite and other various types of sequences and series. The level concludes with topics of limits of functions and continuity and the basics of differentiation.
O	**O 13a** Differentiate the following functions. (2) $y=\ln(x+\sqrt{x^2+1})$ **O 102a** (2) $\int_0^{\frac{\pi}{2}}(2\cos x+\sin2x)\,dx$ **O 154a** 1. When the circle $x^2+(y-a)^2=r^2$ is rotated around the x-axis, the surface generated is called a torus. Determine its volume when $0<r\le a$. [Sol] The circle $x^2+(y-a)^2=r^2$ is formed by an upper and lower part, whose equations are: $y=a+\sqrt{\quad}$ …①, and $y=a-\sqrt{\quad}$ …②, respectively.  Letting V be the volume of the torus, **O 165a** Letting L be the length of the curve $x=g(t)$, $y=h(t)$ (where $a\le t\le b$), $L=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt=\int_a^b\sqrt{\{g'(t)\}^2+\{h'(t)\}^2}\,dt$ In each exercise, determine the length of the given curve. (1) $\begin{cases}x=-e^t\sin t\\y=-e^t\cos t\end{cases}\quad(0\le t\le1)$	Concepts learned through Level N are expanded and reinforced. Students first study advanced differentiation and applications of differential calculus. Students then continue with an in-depth study of advanced integration and its applications. The level concludes with the study of differential equations.
X	In Level X, students can choose to study elective topics. Students can study triangles, vectors, matrices, mapping and transformations, probability and statistics. **XV 67a** 1. Given the fundamental vectors $\overrightarrow{e_1}=(1,0,0)$, $\overrightarrow{e_2}=(0,1,0)$ and $\overrightarrow{e_3}=(0,0,1)$, evaluate the following inner products. (1) $\overrightarrow{e_1}\cdot\overrightarrow{e_2}=$ (2) $\overrightarrow{e_2}\cdot\overrightarrow{e_3}=$ (3) $\overrightarrow{e_3}\cdot\overrightarrow{e_1}=$  2. Given that the cube below has side length a, evaluate the following inner products. (1) $\overrightarrow{AB}\cdot\overrightarrow{AF}=$ (2) $\overrightarrow{DE}\cdot\overrightarrow{BG}=\overrightarrow{DE}\cdot\overrightarrow{A\square}$ =  **XM 32a** 1. Given that A, B and C are all 2×3 matrices, where $B=\begin{bmatrix}1&8&-7\\-2&0&3\end{bmatrix}$, and given that O is a zero matrix, determine the matrices A and C which satisfy the following equations. $3A-4B+2C=O$ $2A-3B+C=O$ 2. Given that $A=\begin{bmatrix}5&0&-1\\-3&1&-3\end{bmatrix}$ and $B=\begin{bmatrix}0&5&7\\1&-2&1\end{bmatrix}$ determine the matrix C which satisfies the following equation. $2C-A=\frac{1}{3}[2B-(4A-C)]$ **XP 30a** 1. Given the letters a,a,a,b,b,b,c,c and c, determine the number of different permutations of lining up the letters. 2. Given the numbers 1,2,3,4,5 and 6, determine how many different three-digit numbers can be formed, allowing repetitions. **XS 26a** (1) Determine the value of k. [Sol] $f(x)=\begin{cases}\frac{k}{2}x & (0\le x\le2)\\ \square & (2\le x\le4)\end{cases}$  $\therefore\int_0^4 f(x)=\int_0^2\frac{k}{2}x\,dx+\int_2^4\square\,dx$ = (2) Evaluate $P(1\le X\le3)$. (3) Determine $\mu(X)$.	

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